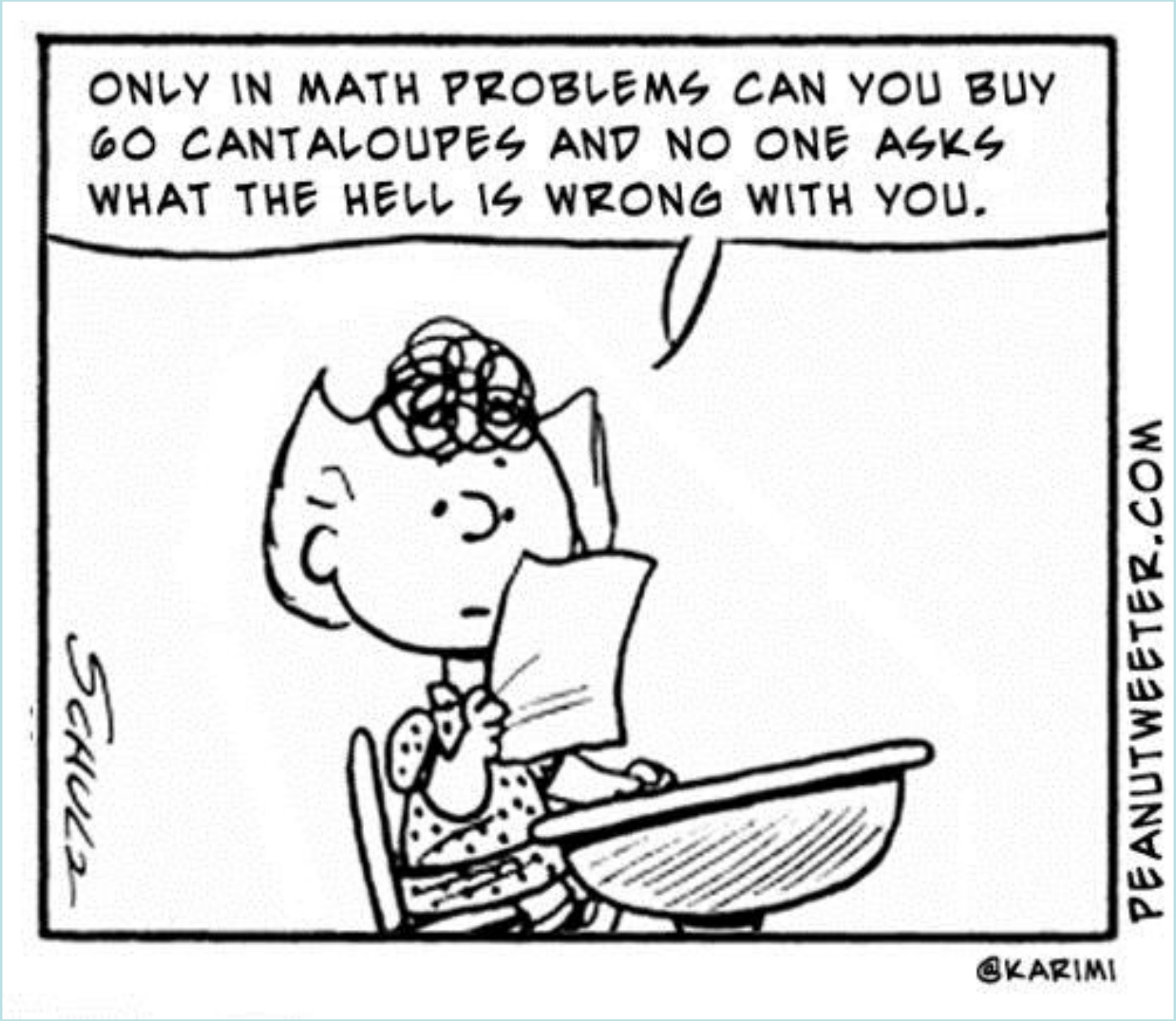
“Mathematical Word Problems as a Specific Genre: Strategies to enable learners of EAL to comprehend and solve word problems.”

NALDIC Conference 2013

Steve Cooke


ONLY IN MATH PROBLEMS CAN YOU BUY 60 CANTALOUPES AND NO ONE ASKS WHAT THE HELL IS WRONG WITH YOU.
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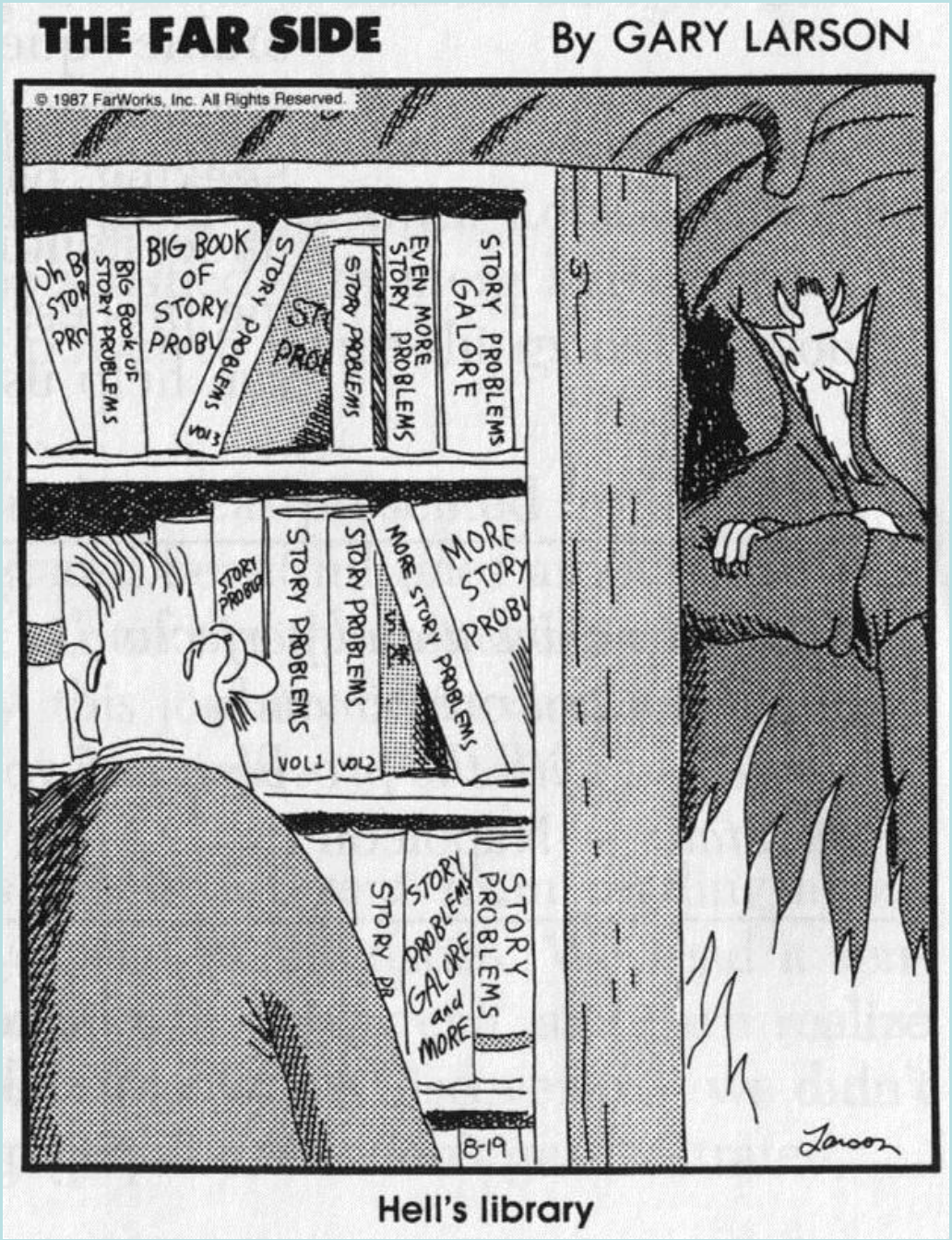
THE FAR SIDE
By GARY LARSON
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STORY PROBLEMS GALORE
EVEN MORE STORY PROBLEMS
STORY PROBLEMS
STORY PROBLEMS VOL 3
BIG BOOK OF STORY PROBL
BIG Book of STORY PROBLEMS
MORE STORY PROBL
MORE STORY PROBLEMS
STORY PROBLEMS VOL2
STORY PROBLEMS VOL1
STORY PROBLEMS GALORE and MORE
STORY PROBLEMS
8-19
Larson
Hell's library

Are word problems stories?

Often;

- No attempt to engage the reader with participants or situations
- No plot
- No motives / reasons / causes / effects
- No new or interesting information
- No reality or perhaps sanity.

Gerofsky

- set up (participants, situation, location etc.)
- the information needed to solve the problem
- a question
- she characterises this as the structure of a mathematical algorithm rather than a ‘story’ and suggests that the setup is irrelevant.
- word problems only relate to real people, objects and actions in an arbitrary way.

Gerofsky (1996)

Gerofsky

- Every year (but it has never happened), Stella (there is no Stella) rents a craft table at a local fun fair (which does not exist). She has a deal for anyone **who** buys more than one sweater (we **know** this to be false). She reduces the price of each additional sweater (and **there** are no sweaters) by 10% of the price of the previous sweater that the person bought (and there are no people, or sweaters, or prices) . . .

Gerofsky (1996)

Assumptions

- that it can be solved mathematically
- that it contains all the necessary information for the solution
- that there is a single right answer
- that the text can be reduced to a mathematical form

Gerofsky (1996)

Tense in word problems

- a lack of correspondence between linguistic tense (L tense), which is indicated by grammatical forms of verbs and metalinguistic tense (M tense) which signals the temporal relation of events relative to the coding time (CT).

- She concludes that word problems are ‘M tenseless’ because the situations described are hypothetical rather than ‘real’ and therefore there is an arbitrary use of L tense which includes a use of the simple present tense even when a chronological sequence of events is being described.

Gerofsky (1996)

Other features of word problems

- Arbitrary and sometimes unusual contexts and vocabulary
- Brevity and density
- Use of ellipsis
- Use of the passive
- Sometimes grammatically incomplete
- Use of hyponymy and superordinates

Some word problems.

- There are 827 trees in a forest. 236 (trees) are cut down. How many trees are still in the forest?
- There are 36 ducks and 35 swans on a lake. How many birds (are there) altogether?
- Newcastle's attendance was 38,387 and West Ham's attendance was 28,723. What was the difference?

A problem

- Agata had 36 books. She had 15 more books than Maria. Then Maria gave 6 of her books to her friend. How many books does Maria have now?

- Why did a Year 5 more advanced EAL learner get the answer 45 and not 15.

An explanation

Agata had 36 books so I put 36 here. She had 15 more books than Maria then I put 15 because she had 15 more. That equals 51. I put 51 here and then Maria gave her 6, 6 of her books to her friend. How many does Maria have now? I put take away 6 because she gave it away the books equals 45.

(Why did you add 36 and 15?)

Because it says Agata 36 books she had 15 more books than Maria. More means that you have more things so she had 15 more so I put 15 here.

Key words

- *Add because more – more means adding so we learnt that more means adding –when we learnt some words – subtract is take away and more means adding.*

(Pupil B. Pr 10).

- *I put add because it was give.*

  Because he lost 9 of his pencils – then he had some pencils – it has the word lost – that means take away.

(Pupil F, Pr. 3 and Pr. 5)

Mathematical ambiguity of key words?

- more
- fewer / less
- altogether
- increase
- decrease
- give
- leave

Addition or subtraction?

- Jozef had 10 apples. He had 7 more than Maria. How many did Maria have?
- Farida and Alex had 15 stickers altogether. Farida had 9 stickers. How many stickers did Alex have?
- Miguel had some sweets. He gave his friend 6 sweets. Now Miguel has 13 sweets. How many sweets did he have in the first place?

Focus of the research

To explore;

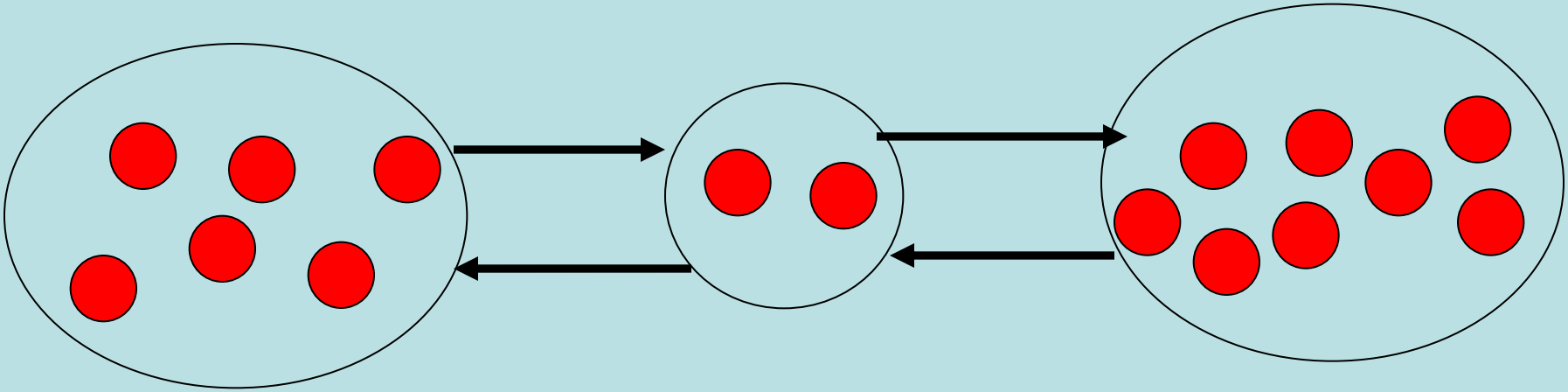
- What the characteristics are of an effective programme of learning activities for improving EAL learners' ability to solve one-step additive word problems.

Theoretical underpinnings

Three key ideas;

- Word problems can be classified according to their semantic structure.
- There is a generally accepted order of relative difficulty of additive word problems
- Children need to build different schemata to relate natural language to the mathematical thinking necessary to solve the problem.

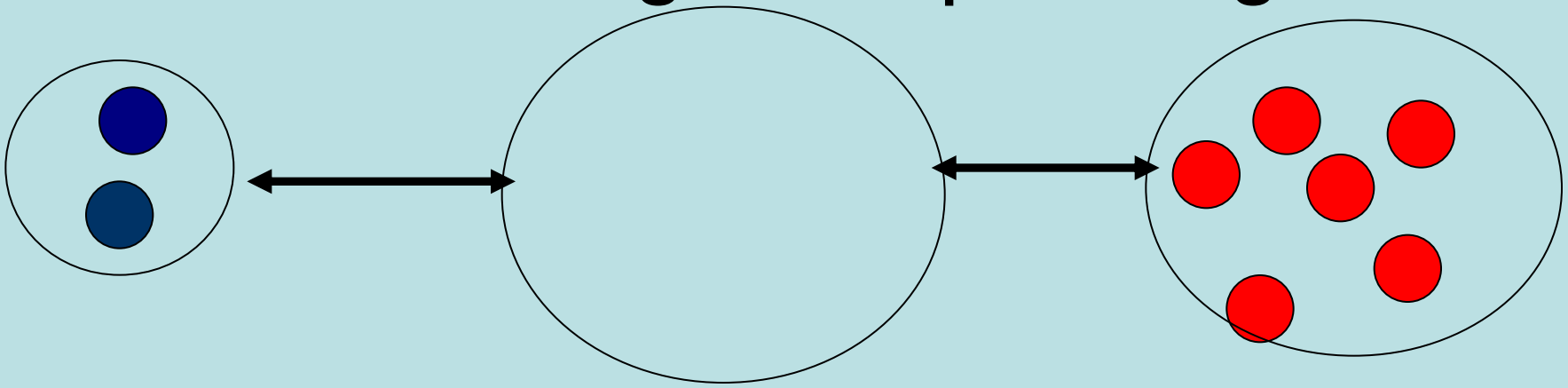
Augmenting or Decreasing



$$6 + 2 = 8$$

$$8 - 2 = 6$$

Combining or Separating

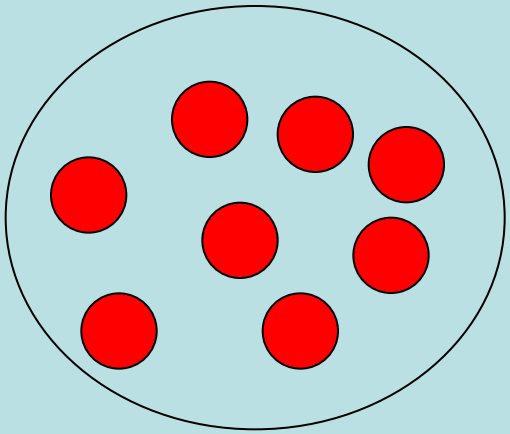


subset superset subset

$$6 + 2 = 8$$

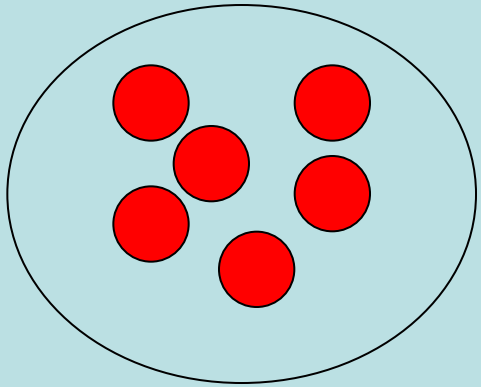
$$8 - 2 = 6$$

Comparing



compared set

difference



reference set

8 – 6 = 2

8 – 2 =6

6 + 2 = 8

Change problems

Alex had 19 stickers. Anna gave him 9 more. How many stickers does Alex have now?	**Change increase Unknown: result set augmenting**	**19 + 9 = ? 9 + 19 = ?**
Maria had 35 sweets. She gave 16 sweets to the children in her class. How many sweets does she have left?	**Change decrease Unknown: result set decreasing**	**35 – 16 =? 16 + ? = 35**
Simon had 35 pence. His brother gave him some money. Now Simon has 49 pence. How much money did his brother give him?	**Change increase Unknown: change set augmenting**	**49 - 35 =14 35 + ? = 49**
The cake shop had 36 fruit cakes. It sold some of them. Now it has 19 fruit cakes. How many did it sell?	**Change decrease Unknown: change set decreasing**	**36 – 19 = ? 36 - ? = 19 19 + ? = 36**
Wiktoria bought 16 books. Now she has 29 books. How many did she have before she went to the shop?	**Change increase Unknown: initial set augmenting**	**29 - 16 =13 ? + 16 =29**
Asif lost 9 of his pencils. Now he has 17 pencils. How many pencils did he have before?	**Change decrease Unknown: initial set decreasing**	**17 + 9 = ? ? – 9 =17**

Combine and Compare

Jane has 7 red T-shirts and 12 blue T-shirts. How many T-shirts does she have altogether?	Combine Unknown: superset combining	7 + 12 = ? 7 + 12 = ?
There are 31 children in the school chess club. 14 of them are boys. How many of them are girls?	Combine Unknown: subset separating	31 – 14 = ? 14 + ? = 31
There are 28 children in Class A. There are 19 children in Class B. How many more children are there in Class A than Class B?	Compare: more Unknown: difference set comparing	28 – 19 = ? 19 + ? = 28 28 - ? = 19
There are 27 children in the sports club. There are 21 children in the art club. How many fewer children are there in the art club than the sports club?	Compare: fewer Unknown: difference set comparing	27- 21 = ? 21 + ? = 27 27 - ? = 21

Compare

Sarah has 16 football stickers. Andre has 26 more stickers than Sarah. How many stickers does Andre have?	Compare: more Unknown: compared set	16 + 26 = ? ? – 16 = 26
Paul has 32 pencils. Mary has 15 fewer pencils than Paul. How many pencils does Mary have?	Compare: fewer Unknown: compared set	32 -15 = ? 15 + ? = 32
Jessica has 16 books. She has 28 fewer books than David. How many books has David got?	Compare: fewer Unknown: ref set comparing	28 + 16 = ? ? - 16 = 28
Farzana has 25 computer games. She has 14 more games than Stefan. How many games has Stefan got?	Compare: more Unknown: ref set comparing	25 – 14 = ? ? + 14 = 25

Order of difficulty?

Change – result unknown - increase
Change - result unknown - decrease
Combine – superset unknown
Combine – subset unknown
Change – change unknown - increase
Change – change unknown - decrease
Compare – difference unknown - more
Compare – difference unknown - fewer
Change – initial unknown –increase
Change – initial unknown – decrease
Compare – compared set unknown – more
Compare –compared set unknown – fewer
Compare – reference set unknown – more
Compare –reference set unknown – fewer

Nesher et al. 1985

Deep structure and surface structure. (Skemp, 1982)

Expressed and contextualised

In language

2 + 6 = 8, 6 + 2 = 8, 8 – 6 = 2, 8 - 2 = 6

Kintsch: Learning from text.(1986)

- Surface structure

Words and phrases encoded in the mental representation but not the meaning

- Textbase

The meaning of the text – the mental representation of the text that a reader or listener constructs in the process of comprehension.

- Situation model

A construction that integrates the textbase and relevant aspects of the reader's knowledge

One Step Word Problem Schema

Thus, a simple word problem, both in additive and in multiplicative structures, is a three component relation R (a, b, c), where a, b, and c are the elements of the problem. The pupil's mental interpretation of this abstract three-place relation, and the action associated with an additive or multiplicative relation comprise the schema of the problem situation.

(Christou and Philippou 1999)

Relation between Language and Schema

The process of constructing a problem representation involves mapping the verbal statement onto an existing schema. Thus, the schema constitutes a vehicle for the comprehension of the semantic relations underlying a given text and its mathematical structure, and it serves as a generalised frame for action in a given situation.

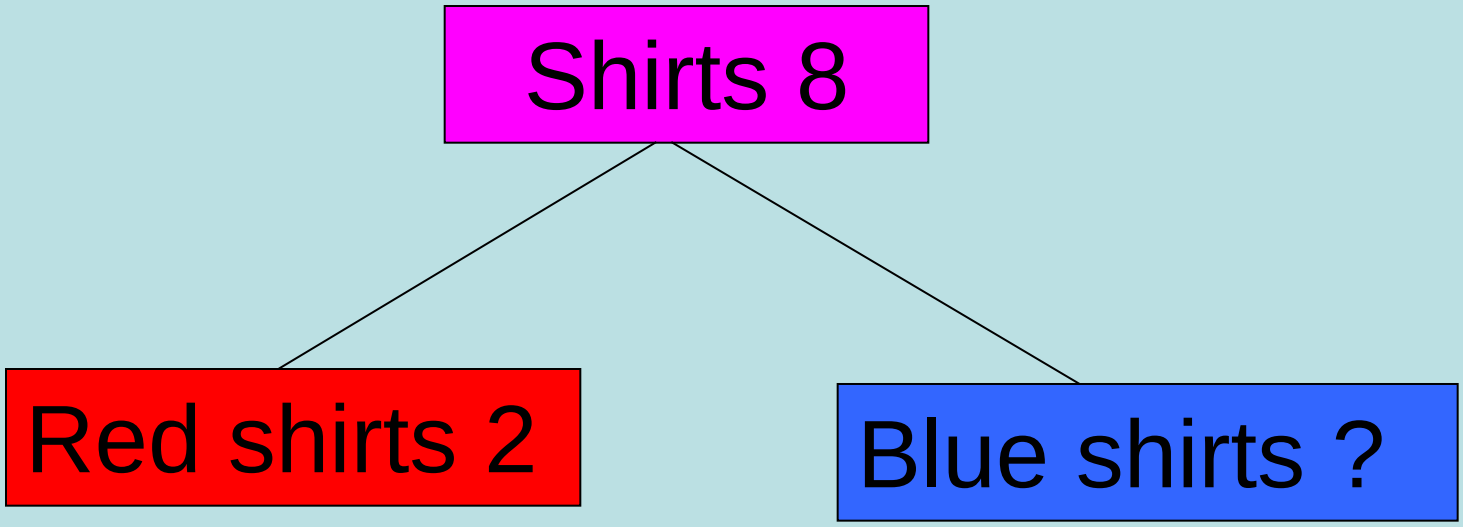
(Christou and Philippou 1999)

Change

start	change	result
books 6	books 2	books ?

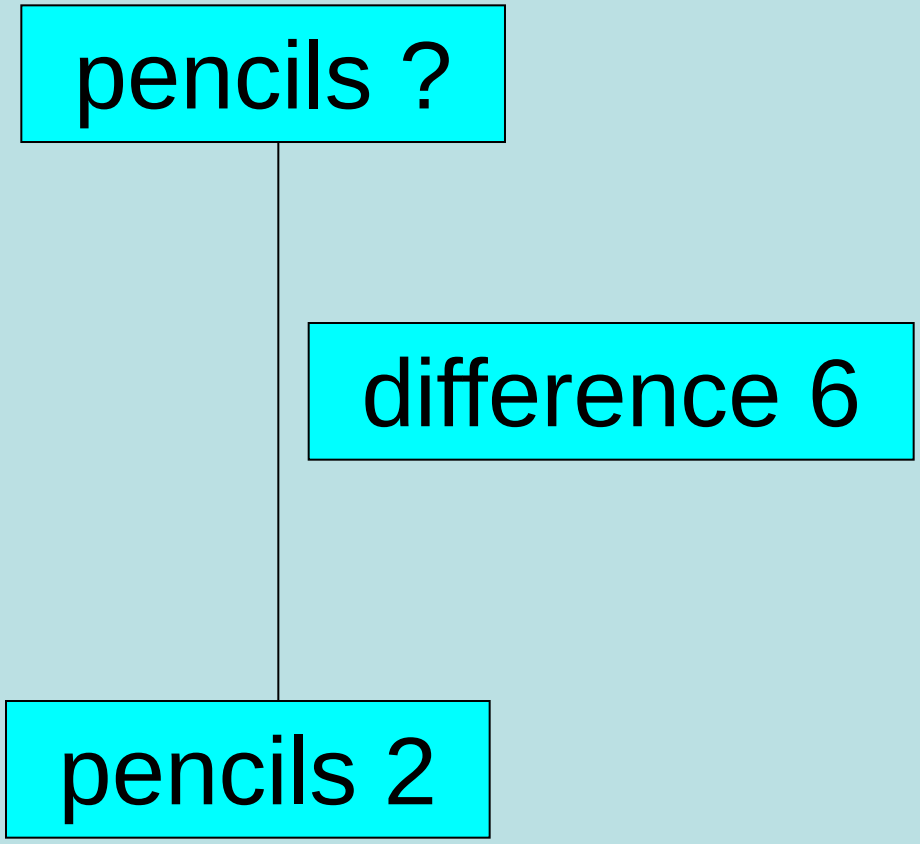
Allia had six books. She bought two books in the shop. How many books has she got now?

Combine



Aftab had eight T shirts. All his T shirts were red or blue. Two them were red. How many of them were blue?

Compare



Anya has six more pencils than Maria. Maria has two pencils. How many pencils does Anya have?

The Pupils Year 4

Pupil	Ethnicity	Language	EAL level	Maths level
A female	Roma	Slovakian	1T	2c
B male	Sinhalese	Sinhalese	2	2c
C female	Pakistani	Panjabi	2	2b
D female	Portuguese	Portuguese	2	1a
E female	Roma	Slovakian	1S	2b
F female	Pakistani	Panjabi	2	2b

Pre test summary

Type	Op	attempted	Operation Correct
Ch inc un= res	+	6	6
Ch inc un =ini	–	6	3
Ch inc un=ch	–	5	1
Ch dec un= res	–	6	5
Ch dec un =ini	+	6	5
Ch dec un=ch	–	6	5
Comb un= sub	–	6	2
Comb un= super	+	6	6
Comp more un= dif	–	6	2
Comp more un= ref	–	6	1
Comp more un= com	+	4	4
Comp fewer un= dif	–	4	1
Comp fewer un= com	–	3	1
Comp fewer un= ref	+	3	3
		73 (25+) 84 (30)	45 (24+)

Data collection

- collect the results of the pre-test.
- audio-record interviews with pupils about their pre-tests.
- audio-record meetings with the EAL teacher
- make notes in meeting with the EAL teacher
- audio-record pupil interactions in lessons.
- make observations of pupils in lessons.
- collect copies of pupils' work
- collect the results of the post-test
- audio-record interviews with pupils about their post test.

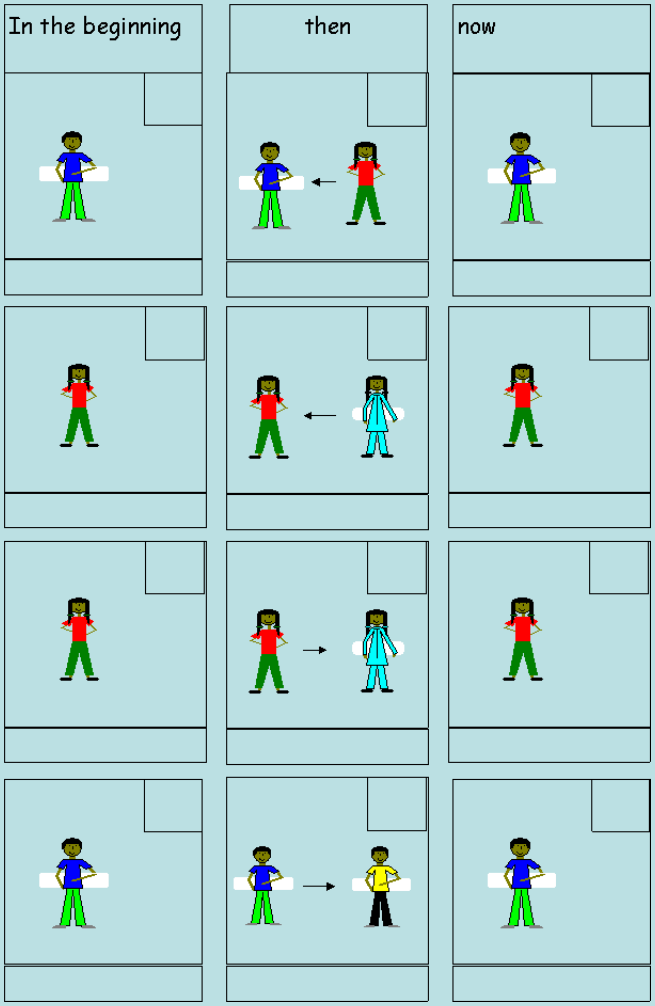
Types of word problem for initial work

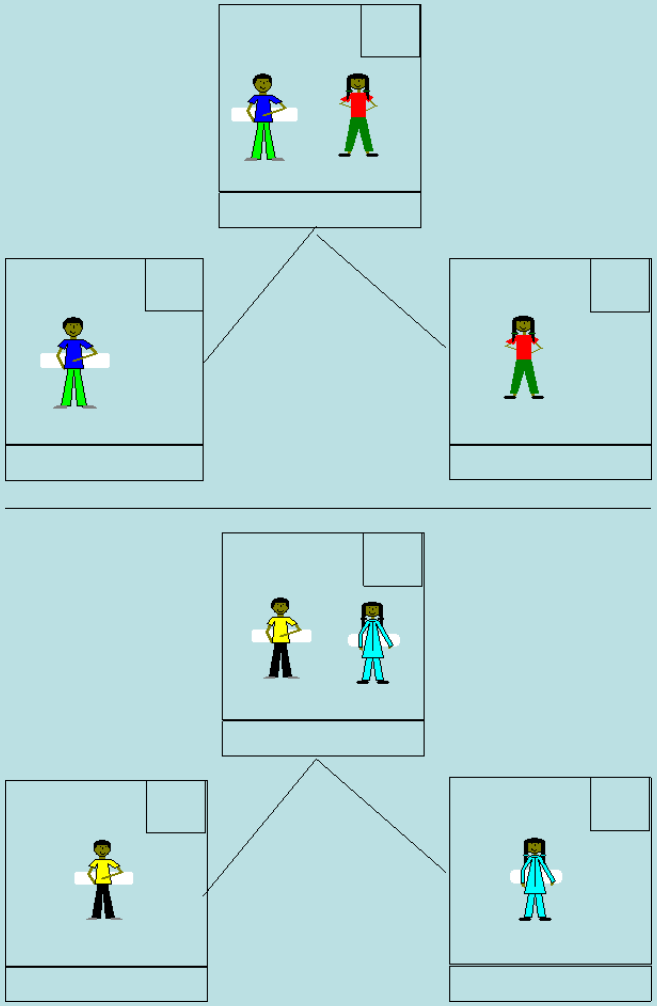
- Change increase unknown result
- Change increase unknown change
- Change decrease unknown result
- Change decrease unknown change
- Combine unknown superset
- Combine unknown subset

Procedure

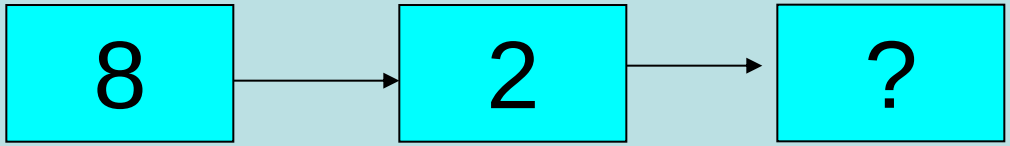
- Role plays using the pupils and real objects.
- Pupils followed the modelled commentary from the teacher, carried out the actions.
- The pupils responded to the question posed.
- After getting used to the role plays the role plays were then mapped on the visual representation.

Visual representations and pictures.

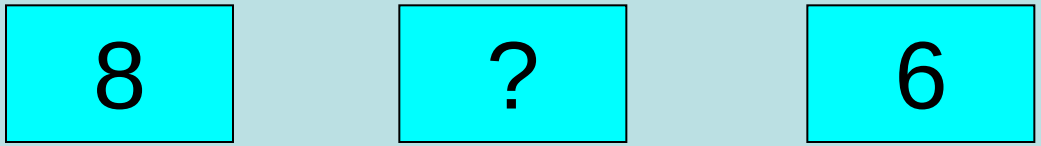




Mapping problems onto visual representations



- Alex had eight biscuits. He gave two biscuits to Alex. How many biscuits does he have now?



- Ayesha had 8 scarves. She gave some of them to her sister. Now she has 6 scarves. How many scarves did she give her sister?

Transform activities

- Change the objects

Maria had 15 _____. Then Maria gave Farida 4 _____. How many _____ does Maria have now?

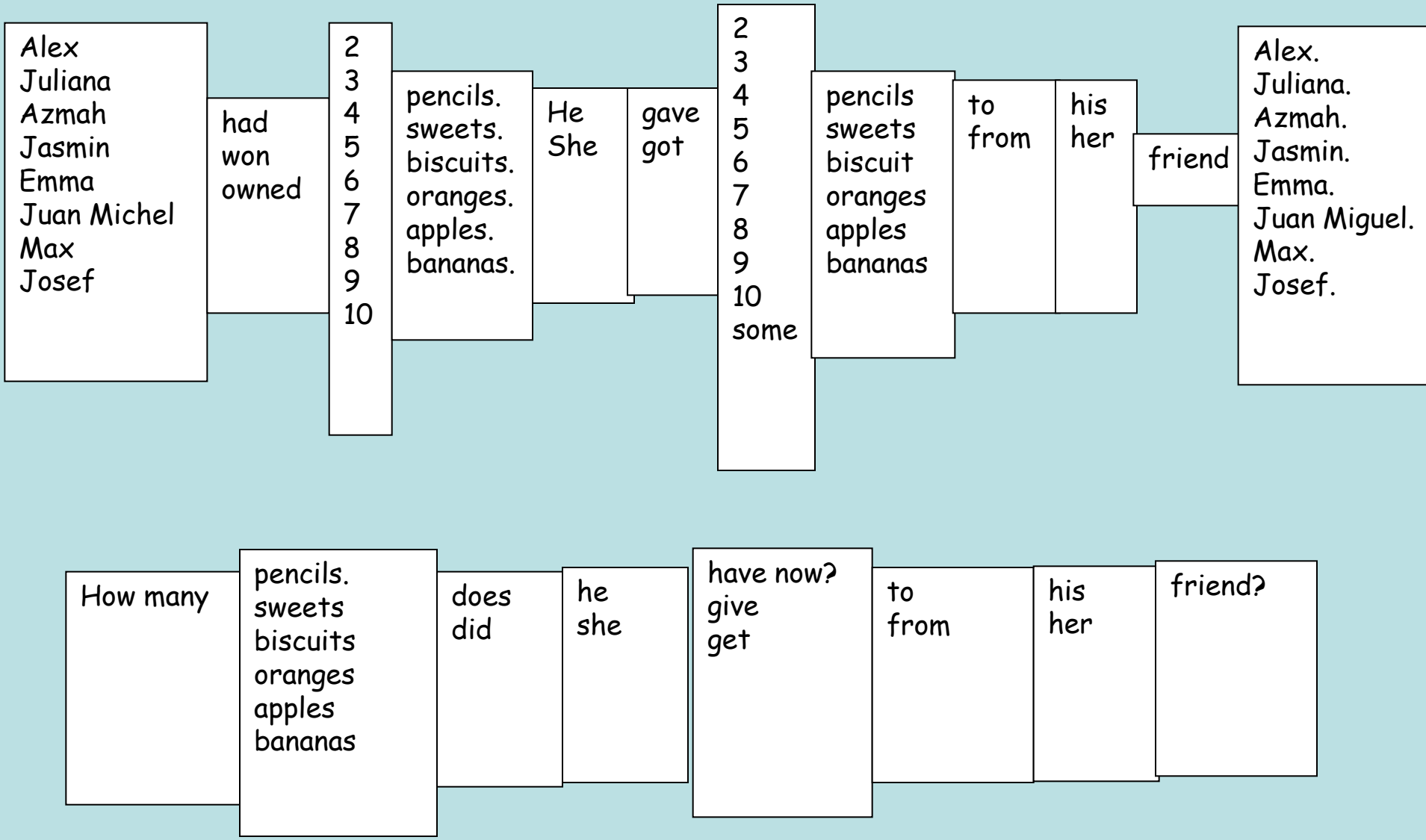
- Change the participants

_____ had 9 sweets. Then _____ gave her some sweets. Now _____ has 13 sweets. How many sweets did _____ give her?

- Change the numbers.

Hussain had ____ sweets. Then Hussain gave Alex some sweets. Now Hussain has ____ sweets. How many sweets did he give Alex?

Change Word problems



Results

Over the sequence of lessons we were able to introduce 10 problem types:

- Change problems result and change unknown
- Combine problems superset and subset unknown
- Compare problems difference and compared set unknown

Pupils

- All of them improved their scores on the post test for the ten word problem types covered in the sessions.
- Four of the pupils solved all of the problems correctly
- One pupil made a calculation error.
- One pupil made an operation error.
- All of them felt that they had improved in their maths and problem solving.

Observations suggested that pupils

- were not relying on addition as the default operation
- were reading the whole problem before starting the problem solving process
- used the visual representations initially but were able to do without them as they got used to the problem type
- were confident in reading the word problem?

Pupils;

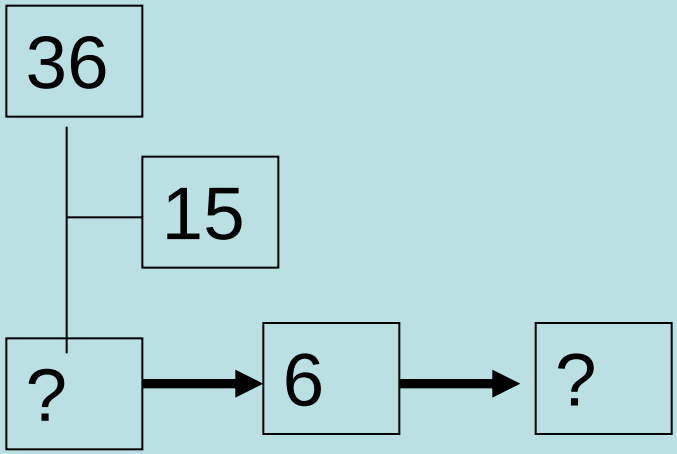
- were able to construct coherent word problems
- used key words not to determine operations but to determine the schema which then helped them to determine the appropriate operation.
- improved their number bonds and speed and accuracy of calculations and working with word problems as beneficial in this process.

Next steps for the pupils?

- further work on Change initial unknown and Compare unknown reference set problems
- continue to provide opportunities for pupils to construct their own word problems.
- practice with known problem types with more varied vocabulary and syntax
- introduce two-step problems using combinations of the visual representations

Two step problems.

- Agata had 36 books. She had 15 more books than Maria. Then Maria gave 6 of her books to her friend. How many books does Maria have now?



Next steps for the school

- Use role play and visual representations to support understanding of word problems particualrly in KS 1
- Introduce word problems to pupils by type rather than by operation
- Be cautious about associating ‘key words’ with specific operations.
- Include problem transformation and pupil construction of word problems in any cycle of activities.

Next steps more widely

- Trial these materials and approaches in other schools and circumstances
- Continue to investigate the relation between natural language, visual representations and mathematical understanding
- Frame further investigations in terms of ‘learning trajectories’ which take into consideration language development and mathematical cognition.

Further reading

WHAT WORKS?

Research into Practice

A research-into-practice series produced by a partnership between The Literacy and Numeracy Secretariat and the Ontario Association of Deans of Education

Research monograph 34

Word problems: Connecting language, mathematics and life

By Dr. Richard Barwell

University of Ottawa, Faculty of Education

http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/whatworks.html

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